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Variation of temperature and density dependent screened potential between two charges and static pair correlation function of electrons around a positron in two dimensional weakly coupled one component plasma

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Abstract : Screened potential between two charges and the static pair correlation function of electrons around a positron in two dimensional one component classical and quantum weakly coupled plasmas at different temperatures, 0.1 K $\leq T \leq$ 5 K, have been studied using the recently suggested wave vector and frequency dependent complex dielectric functions of the plasmas.

Keywords : Surfaces and interfaces, screened potential, static pair correlation function.

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l. Introduction

)bservation of quantum Hall effect, both integral [1] and ractional [2] in two-dimensional (2D) degenerate electron as at very low temperatures and the evidence of ccurrence of Wigner solid [3,4] in strongly coupled lassical 2D electron gas on the surface of clean liquid le, *etc*., point towards the unique physical features that an arise in 2D one-component plasmas. As in three limensions, in 2D too, one can very the concentration of lectrons n, expressed as a real number density, ranging rom ideal non-interacting situation to highly degenerate, lepending upon the value of $n\lambda_{th}^2$. Here, $\lambda_{th} = h/\sqrt{2mk_BT}$ is the thermal de-Broglie wave length, ı is the Planck's constant, k_B is the Boltzmann's constant, n is the mass of an electron and T is the temperature. While $n\lambda_{th}^2 << 1$ ensures that the system behaves lassically, $n\lambda_{th}^2 > 1$ describes the quantum 2D electron ystem. The competition between the Coulomb potential energy and thermal energy per electron given by $\Gamma = (\pi n)^{1/2}e^2/k_BT$, e being the electronic charge, determines essentially the strength of coupling. When $\Gamma < 1$, the coupling is weak, otherwise it is intermediate or strong. Though considerable effort has been made recently to understand the strongly coupled, both classical [5,6] and quantum [7] 2D one-component plasma, there has been little effort to understand the behaviour of weakly coupled one-component classical [8] and quantum plasmas, which can be realized experimentally, amongst other methods, on the surface of liquid He [9,10], when electrons of areal number density, $10^5 \leq n \leq 10^{10}$ cm^{-2} are trapped.

Recently, unlike earlier studies [8], complete expressions for the wave vector, q and frequency ω dependent complex dielectric function for weakly coupled 2D one-component classical and quantum plasmas [11] have been obtained and used to study the dynamical structure factor which yields the full collective equilibrium dynamics of the plasma at a given temperature. In the present communication, we have described the screened

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potential betweent the two charges and the static pair correlation functions of electrons around a positron located in the two-dimensional, one-component classical plasma. To our knowledge no such studies have been reported so far.

2. Mathematical formalism

Screened potential between two charges separated by a distance r [12], in a two-dimensional, one-component plasma is given by

$$V_{sc}(r)=\frac{1}{(2\pi)^2}\int e^{iq\cdot r}V_{sc}(q)dq\,. \tag{1}$$

Here,

$$V_{sc}(q)=\frac{2\pi e^2}{q\varepsilon(q,\omega=0)}\,. \tag{2}$$

$\varepsilon(q,\omega=0)$ is the zero frequency dielectric function which can be obtained from the recently reported wave vector and frequency-dependent complex dielectric function which for quantum one-component plasma is given as [11,13,14]

$$\varepsilon^Q(q,\omega)=\varepsilon_1^Q(q,\omega)+i\varepsilon_2^Q(q,\omega)$$

$$=1+\sqrt{2}\frac{\omega_p^2}{q^2v^2}\frac{mv}{hq}\left[e^{-\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^2}\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)\right.$$

$$\times\left(1+\frac{1}{3}\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^2+\ldots\right)$$

$$\times-e^{-\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^2}\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)$$

$$\left.\times\left(1+\frac{1}{3}\left(\frac{\omega}{\sqrt{2}qv}-\frac{hq}{2\sqrt{2}mv}\right)^2+\ldots\right)\right]+i\sqrt{\frac{\pi}{2}}\frac{\omega^2}{q^2v^2}\frac{mv}{hq}$$

$$\times\left[e^{-\left(\frac{\omega}{\sqrt{2}qv}-\frac{hq}{2\sqrt{2}mv}\right)^2}-e^{-\left(\frac{\omega}{\sqrt{2}qv}+\frac{hq}{2\sqrt{2}mv}\right)^2}\right], \tag{3}$$

where $\omega_p=\left(\sqrt{\frac{2\pi ne^2q}{m}}\right)$ is the q-dependent 2D plasma frequency. $v=\left(\sqrt{\frac{k_BT}{m}}\right)$ is the thermal velocity. When h is put equal to zero the above expression reduces to the corresponding expression for classical plasma

$$\varepsilon^{cl}(q,\omega)=1+\frac{\omega_p^2}{q^2v^2}-\frac{\omega_p^2}{q^2v^2}\frac{\omega^2}{q^2v^2}e^{-\omega^2/2q^2v^2}\left(1+\frac{1}{3}\left(\frac{\omega_p^2}{2q^2v^2}\right)\right.$$

$$\left.+\frac{1}{10}\left(\frac{\omega_p^2}{2q^2v^2}\right)^2+\cdots\right)+i\sqrt{\frac{\pi}{2}}\left(\frac{\omega_p^2}{q^2v^2}\right)\frac{\omega}{qv}e^{-\omega^2/2q^2v^2}\,. \tag{4}$$

$\varepsilon(q,\omega=0)$ from expression (3) to the order h^2 is as follows :

$$\varepsilon^Q(q,0)=1+\frac{a}{q}-\frac{1}{12}\frac{ah^2}{mk_BT}q\,, \tag{5}$$

$$\text{where, } a=\frac{2\pi ne^2}{k_BT} \tag{6}$$

when $h=0$, the expression (5) reduces to $\varepsilon^{cl}(q,0)$.

Substituting expression (5) in expression (2) and then solving expression (1), keeping in mind the two-dimensional nature of the integral, one gets the following expression :

$$V_{sc}(r)=e^2\int\frac{qJ_0(qr)}{q+a}dq+\frac{e^2ah^2}{12mk_BT}$$

$$\int J_0(qr)\frac{q^3}{(q+a)^2}dq, \tag{7}$$

where $J_0(qr)$ is the zeroth order Bessel function.

Second term in expression (7) is the first quantum mechanical correction term to the classical expression which is given by its first term. Using the following expansion of $J_0(qr)$ [15],

$$J_0(qr) = \sum_{k=0}^{\infty} \frac{(-1)^k (qr)^{2k}}{2^{2k}(k!)^2}. \tag{8}$$

The classical expression for the screened potential $V_{sc}(r)$ upto the order of r^4 in $J_0(qr)$ is given as

$$V_{sc}(r) = \frac{e}{\epsilon} \; 1 - ar\log\left(\frac{q'+a}{a}\right) - \frac{r^3a^3}{4}\log\left| \; q'+a \right.$$

$$\frac{-r^3a}{4}\left(\frac{q'^2}{2} - aq'\right) - \frac{ar^5}{64}$$

$$\left(\frac{q'^4}{4} + \frac{a^2q'^2}{2} - \frac{aq'^3}{3} - a^3q'\right) - \frac{a^5r^5}{64}\log\left(\frac{q'+a}{\quad}\right. \tag{9}$$

$$= \frac{e}{\epsilon} f_{sc}(r), \tag{10}$$

where q' is some large value of q which corresponds to infinity and $f_{sc}(r)$ is the screening function.

The static pair correlation function of electrons $g^{\pm}(r)$ [16–18] around a finite mass, positively-charged impurity in a two-dimensional plasma is given by the expression

$$g^{\pm}(r) = 1 + \frac{1}{(2\pi)^2}\int_0^{\infty} f(r) e^{i\mathbf{q}\cdot\mathbf{r}} dq, \tag{11}$$

where $f(q)$, the static structure factor is the Fourier transform of $g^{\pm}(r)$ and is given as

$$f(q) = -\frac{z}{n}\mathrm{Re}\left(\frac{1}{\varepsilon\left(q, \hbar q^2/2M\right)}\right)$$

$$+\frac{z}{n}\frac{2}{\pi}\int_0^{\infty} d\omega \, \mathrm{Im}\left(\frac{1}{\varepsilon(q,\omega)}\right) \times P\left(\frac{\hbar q^2/2M}{\omega^2 - (\hbar q^2/2M)^2}\right), \tag{12}$$

where M is the mass of positively charged impurity taken to be positron in the present problem and z is a unit positive integer. Solving the two-dimensional integral in expression [11], we get

$$g^{\pm}(r) = 1 + \frac{1}{(2\pi)}\int_0^{\infty} f(q) J_0(qr) q dq. \tag{13}$$

Making use of expressions (3) and (4) for quantum and classical dielectric functions respectively, in expression (12) and substituting the resulting expression in expression (13), static pair correlation function of electrons is computed by using the expression (8) of $J_0(qr)$.

3. Results and discussion

It is clear from expression (9) that the screened potential between two electrons in the two-dimensional plasma does not follow the exponential behaviour as is the case in corresponding three-dimensional plasma [12] and is quite involved in terms of its dependence on temperature and density. It may also be pointed out that the terms of higher order in qr in the expansion of $J_0(qr)$ contribute little to $V_{sc}(r)$. The static pair correlation function in the 2D situation is also quite different from the corresponding three-dimensional case [16] and unlike the latter, the plasma frequency is q-dependent and occurs in all the terms of $\varepsilon(q\omega)$ which have to be taken into account in its evaluation.

Computations have been made for $V_{sc}(r)$ using expression (9) on two areal densities of electrons n = 7.65×10^4 cm^{-2} and 7.65×10^5 cm^{-2} and are shown in Figure 1 for temperatures T = 5, 1 and 0.1 K. For these densities and temperatures, $n\lambda_{th}^2$ is much less than 1, but the coupling though weak at high temperatures becomes intermediate at the lowest temperature. In the calculations, value of q' is taken to be 2.0×10^4 cm^{-1} which yields the converged result. The variation of q' around this value does not result in any perceptible change in the numerical values. In Figure 1(a), we have shown the variation of $V_{sc}(r)$ with r expressed in terms of r_s, r_s being the radius of the circle assigned to an electron, at 5, 1 and 0.1 K. The areal number denote of electron $n = 7.65 \times 10^4$ cm^{-2}. The potential between the two charges is screened differently at different temperatures and the screening increases with decrease in the temperature of the 2D electron gas. This has been brought out more clearly in the inset of the figure where screening function, $f_{sc}(r)$ *vs* r has been plotted. When $f_{sc}(r)$ is 1 there is no screening as is evident from expression (10). $f_{sc}(r)$'s variation with temperature is quite involved at different temperatures as is clear from the inset of the figure and expression (9). The temperature dependent screening is quite sensitive to even a small change in temperature, indicating the significant effect of kinetic energy. In Figure 1(b), we have shown the similar variations for density

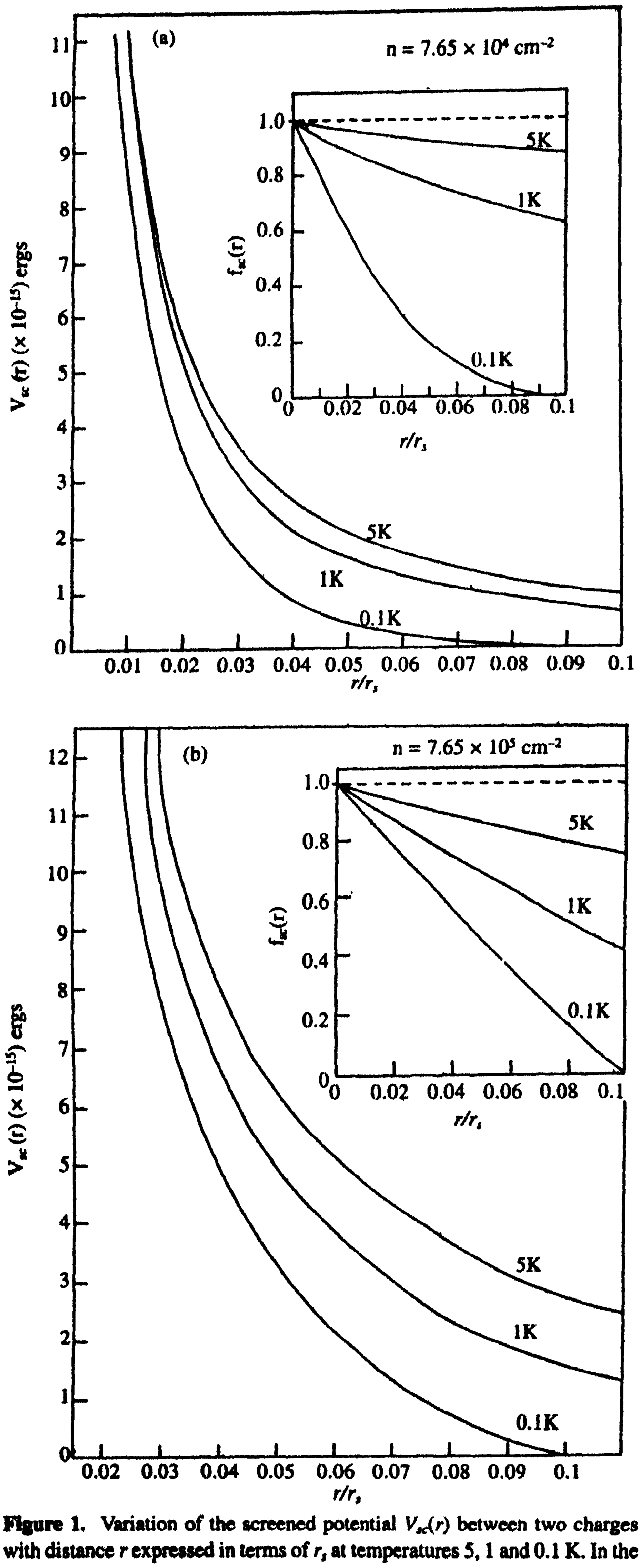

Figure 1. Variation of the screened potential $V_{sc}(r)$ between two charges with distance r expressed in terms of r_s at temperatures 5, 1 and 0.1 K. In the inset, variation of screening function $f_{sc}(r)$ is shown with distance expressed in terms of r_s for the three temperatures. (– – –) line indicates $f_{sc}(r)$ equal to 1. Areal number density (a) $n = 7.65 \times 10^4$ cm^{-2}, (b) $n = 7.65 \times 10^5$ cm^{-2}.

$n = 7.65 \times 10^5$ cm^{-2}. As the density increases, the screening function also changes appreciably as is evident from the Figures 1(a) and 1(b) insets. At higher density, the screening is much more upto 1 K at a given charge separation in comparison with that at lower density. It may be noted that the value of r_s is quite different for the two densities. These results are contrast to the case of highly degenerate 2D electron gas at $T = 0$, where the screening turns out to be independent of areal number density [19]. The second term in expression (7) which yields the quantum mechanical correction to classical $V_{sc}(r)$, as discussed earlier, has also been computed for different cases. However, its contribution is so small even at 0.1 K and higher density that it cannot be shown in the Figures.

The static pair correlation function, $g^{\pm}(r)$ of the electrons around a positron placed in a two-dimensional, one-component plasma specified in the discussion of $V_{sc}(r)$ has been computed for the two areal densities and temperatures ranging from 10 K to 0.1 K using expression (13). Unlike the case for $V_{sc}(r)$, here, the total dielectric function has to be used. The results of the computations have been plotted in Figures 2(a) and 3(a) for the area densities 7.65×10^4 cm^{-2} and 7.65×10^5 cm^{-2} respectively at temperatures 1, 2, 5 and 10 K. $g^{\pm}(r)$ has maximum value for $r = 0$ at all the temperatures which increases rapidly as the temperature is decreased. The variation of $g^{\pm}(r)$ with r expressed in terms of r_s, shows significant undulation at different values of r which decrease with increase in temperature as shown in the Figures. The details of such variation in $g^{\pm}(r)$ for $r/r_s > 0.3$ have been shown in the inset of the Figures. At low values of r, high energy is required to force the particles to overlap, so $g^{\pm}(r)$ is almost zero. At a distance roughly equal to atomic diameter, there is a pronounced peak in $g^{\pm}(r)$, denoting the sphere of nearest neighbour. As r increases, the presence of wiggles denote the sphere of nearest neighbour. Even when $r = r_s$, the wiggles though small in amplitude are still present. As the density is increased to 7.65×10^5 cm^{-2}, these wiggles are considerable reduced and decrease with increase in temperature as is clear from Figure 3(a). The positron annihilation rate which is proportional to $g^{\pm}(0)/r_s^2$ (r_s given in atomic units), decreases with increase in temperature and is larger by almost an order for the higher density in comparison with the lower density. The computations which use quantum mechanical $\varepsilon(q,\omega)$ in (12) and subsequent evaluation of $g^{\pm}(r)$ yield numerical values only slightly different from that of the classical result. Only when the temperature is reduced to 0.1 K does one get noticably different values of $g^{\pm}(r)$ for the two densities at small values of r, as shown in Figures 2(b) and 3(b). In the Figures, while the dashed curve indicates the variation of $g^{\pm}(r)$ for the classical case, while the continuous curve shows the variation when dual nature of the electrons has been considered. The positron annihilation rate in the quantum mechanical case turns out to be some what smaller than that in the corresponding classical situation.

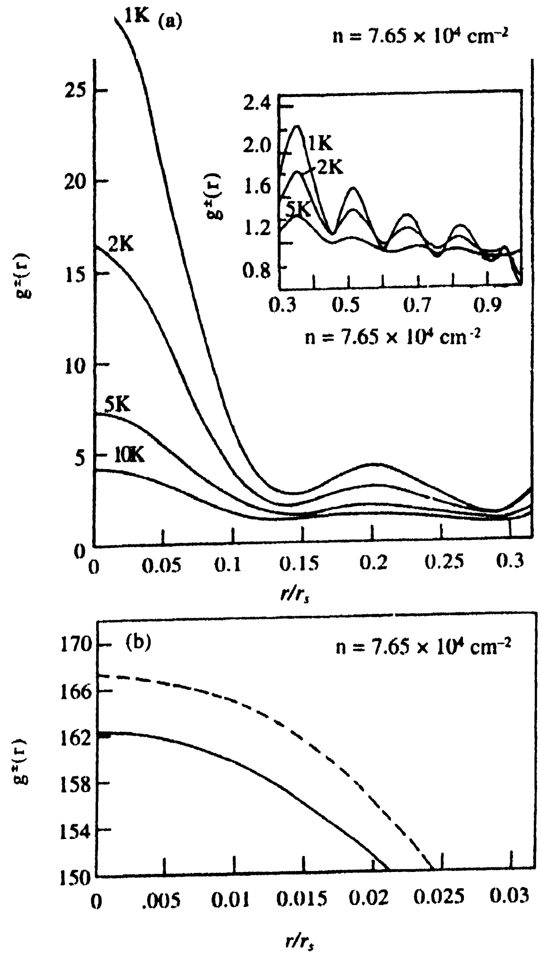

Figure 2. (a) Variation of static pair correlation function $g^{\pm}(r)$ of electrons with distance r expressed in terms of r_s at temperature 10, 5, 2 and 1 K. Areal number density of electrons $n = 7.65 \times 10^4$ cm^{-2}. In the inset of the figure, variation of $g^{\pm}(r)$ is shown for $r/r_s > 0.3$ for 5, 2 and 1 K. (b) Variation of $g^{\pm}(r)$ with destance r expressed in terms of r_s for $T = 0$. 1 K. The dashed curve represents classical plasma and the continuous curve the quantum

4. Conclusions

From the study, one may conclude that both the screened potential and static pair correlation function of electrons in two-dimensional, one-component plasma are quite different and involved. The variation of the screening function depends significantly both on the areal number density of electrons and its temperature. The positron annihilation rate in such plasma can be observed experimentally as these systems have already been realized in practice.

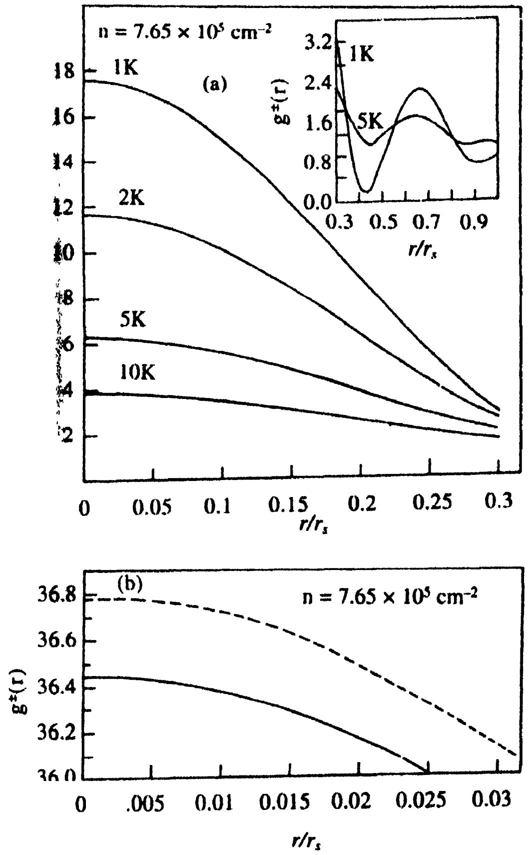

Figure 3. Same as in Figures 2(a) & (b); but the areal number density of electrons is 7.65×10^5 cm^{-2}.

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